Measuring Inflation Expectations Uncertainty Using High-Frequency Data

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Abstract

Inflation expectations play a key role in determining future economic outcomes, and its uncertainty provides a direct gauge of how well-anchored the inflation expectations are. We construct a model-based measure of inflation expectations uncertainty by augmenting a standard unobserved components model of inflation with information from noisy and possibly biased measures of inflation expectations obtained from financial markets. This new model-based measure of inflation expectations uncertainty is better calibrated and provides valuable information for policymakers. Using US data, we find significant changes of inflation expectations uncertainty during the Great Recession.

Keywords: trend inflation, inflation expectations, stochastic volatility

JEL Classification: C11, C32, E31

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1 Introduction

Inflation expectations play a key role in price and wage setting behavior, and therefore have a substantial influence on future economic outcomes. Policymakers and central bankers in particular pay close attention to measures of long-run inflation expectations—these expectations reveal information about the credibility of monetary policy and are an important input in the formulation of macroeconomic policy in general. Given their importance, there is now a large and growing literature on combining econometric models of trend inflation with inflation expectations from surveys of professionals or consumers to obtain better estimates of inflation expectations or better inflation forecasts. Prominent examples include Kozicki and Tinsley (2012), Wright (2013), Nason and Smith (2013) and Mertens (2016).

Building on this line of research, we investigate the information content of market-based measures of inflation expectations for refining estimates of inflation expectations *volatility* or *uncertainty*. Such a measure is useful for several reasons. Monetary policy tools work differently if inflation expectations are firmly anchored than if they are not. In particular, monetary policy is thought to be most effective when inflation expectations are stable. Hence, a measure of inflation expectations uncertainty provides a direct gauge of how well-anchored the inflation expectations are. This measure can be used to assess, for example, the effectiveness of forward guidance. A more refined measure of inflation expectations uncertainty can also be used to develop by a better second moment policy such as a financial stabilization package to reduce systemic risk.

To construct such a measure of inflation expectations uncertainty, we develop a new bivariate unobserved components model. We take a model-based approach and combine direct measures of inflation expectations uncertainty and information in model-based estimates. In essence, we aim to view these direct measures of inflation expectations uncertainty through the lens of an econometric model. Our point of departure is the univariate unobserved components model with stochastic volatility (UCSV) of Stock and Watson (2007) that is widely used to model inflation (see, e.g., Chan, Koop, and Potter, 2013; Clark and Doh, 2014). Under some assumptions, trend inflation from this model should correspond to long-run inflation expectations. Hence, the time-varying volatility of trend inflation can be interpreted as long-run inflation expectations uncertainty.

We augment this model-based measure of uncertainty with information from market-based inflation expectations. Specifically, we obtain breakeven inflation computed from long-horizon real and nominal bonds, which is available daily. We then compute the associated realized volatility (see, e.g., Andersen et al., 2003), say, within a month. The constructed realized volatility gives a quantitative measure of the variation of inflation expectations, but it may also reflect other idiosyncratic factors such as volatility of risk premiums. As such, it may not be appropriate to directly equate the realized volatility with the volatility of inflation expectations. However, we can still incorporate this additional information into the UCSV model by adding a new measurement equation that relates the realized volatility to the latent time-varying volatility of trend inflation. Using this bivariate model, we can extract useful information in the realized volatility to refine estimates of trend inflation volatility.

Using US data, we find that the constructed measure of realized volatility helps improve the estimation precision of inflation expectations uncertainty compared to the benchmark UCSV model. We find significant changes of inflation expectations uncertainty during the Great Recession, in contrast to the largely flat estimates from the UCSV model. By incorporating the realized volatility, the new model is able to pick up drastic changes in inflation expectations uncertainty. Using the marginal likelihood as a model comparison criterion, we show that this new model compares favorably to the benchmark.

Our paper is also related to the literature on measuring uncertainty and studying its impact on the economy. Since the seminal paper by Bloom (2009), many studies have contributed to this literature, including Bloom (2013), Caggiano, Castelnuovo, and Groshenny (2014), Jurado, Ludvigson, and Ng (2015) and Mumtaz and Theodoridis (2015). In particular, Berument, Yalcin, and Yildirim (2009) and Chan (2017) study the impact of inflation volatility on inflation. Our paper provides a new measure of inflation expectations volatility, and it would be interesting in future work to study its impact on other macroeconomic variables.

The rest of this paper is organized as follows. Section 2 introduces the model and discusses the interpretation of trend inflation volatility. Section 3 defines the breakeven inflation and explains how the realized volatility measure is constructed. The data and estimation are outlined in Section 4, and Section 5 presents the results using US inflation data. Finally, Section 6 concludes and discusses some future research direction.

2 Modeling Trend Inflation Uncertainty

The trend-cycle decomposition of inflation, π_t, is motivated by the idea that it can be usefully viewed as the sum of two separate components: a nonstationary component that represents the trend inflation, π_t^*, and a transitory deviation from the trend, or the inflation gap, u_t^π:

$$\pi_t = \pi_t^* + u_t^\pi. \tag{1}$$

To identify the two components, one typically makes assumptions that imply

$$\lim_{j\to\infty} \mathbb{E}_t \pi_{t+j} = \lim_{j\to\infty} \mathbb{E}_t \pi_{t+j}^* = \pi_t^* \tag{2}$$

and

$$\lim_{j\to\infty} \mathbb{E}_t u_{t+j}^\pi = 0, \tag{3}$$

where $\mathbb{E}_t$ is the conditional expectation given the information at time t. For example, if one assumes that π_t^* follows a random walk and u_t^π follows a stationary AR(1) process with 0 mean, then both conditions are satisfied. The decomposition in (1) together with the conditions in (2) and (3) maybe seen as a generalization of the Beveridge-Nelson decomposition (Beveridge and Nelson, 1981).

Under the conditions in (2) and (3), we may view the trend inflation π_t^* as some long-horizon inflation expectation. Specifically, given the information at time t, the expected future inflation for period $t + j$ for some large j should provide an estimate of π_t^*. A few recent papers have exploited this relationship and used survey long-horizon inflation expectations made at time t to produce estimates of current trend inflation, with Kozicki and Tinsley (2012) and Faust and Wright (2013) being prominent examples.

Our modeling approach is similar to the one in Chan, Clark, and Koop (2015), where they extend the unobserved components model with stochastic volatility (UCSV) of Stock and Watson (2007) by incorporating survey-based long-run inflation expectations. More specifically, an additional measurement equation is added to include long-run inflation expectations, x_t, obtained from the Federal Reserve Board of Governor's FRB/US econometric model and Blue Chip Consensus. They find that that long-run inflation expectations can substantially refine estimates of trend inflation over popular alternatives. But it is inappropriate to equate trend inflation with the long-run inflation expectations.

We exploit a different source of information for a different purpose. Specifically, we investigate if the realized volatility of market-based long-horizon inflation expectations at time t—which we denote as z_t—provides useful information for the trend inflation *uncertainty*. Heuristically, because we observe the market-based measure of inflation expectations at daily frequency, we could obtain an estimate of the variance of inflation expectations at lower frequency, say, monthly, as in Andersen, Bollerslev, Diebold, and Labys (2003) and Barndorff-Nielsen and Shephard (2002), and use it as a measure of inflation expectations uncertainty. However, since the theory of realized volatility relies on infill asymptotics—where more data are collected by sampling more intensely in a fixed domain—it may not be reasonable to assume that the realized volatility measure from the daily observations would be consistent for the integrated variation at monthly frequency. But one may still view this measure as a potentially useful source of information inspired by the realized volatility theory. We will delineate data construction in more details in the next section.

We take the UCSV model of Stock and Watson (2007) and augment it with an additional measurement equation of the realized volatility z_t. More specifically, consider the following model:

$$\pi_t = \pi_t^* + u_t^{\pi}, \qquad u_t^{\pi} \sim \mathcal{N}(0, \mathrm{e}^{h_t}), \tag{4}$$

$$\pi_t^* = \pi_{t-1}^* + u_t^{\pi^*}, \qquad u_t^{\pi^*} \sim \mathcal{N}(0, \mathrm{e}^{g_t}), \tag{5}$$

$$h_t = h_{t-1} + u_t^h, \qquad u_t^h, \sim \mathcal{N}(0, \sigma_h^2), \tag{6}$$

$$g_t = g_{t-1} + u_t^g, \qquad u_t^g, \sim \mathcal{N}(0, \sigma_g^2), \tag{7}$$

$$\log z_t = a_0 + a_1 g_t + u_t^z, \qquad u_t^z \sim \mathcal{N}(0, \sigma_z^2), \tag{8}$$

where h_t and g_t are respectively the log volatility of the transitory and trend components.

The UCSV model of Stock and Watson (2007) is defined by (4)–(7). The new equation is (8), which relates the log realized volatility measure to g_t, the log volatility of the trend inflation. Since z_t is likely to be a noisy and potentially biased measure of trend inflation volatility, the measurement equation (8) allows us to estimate the relationship instead of imposing equality. For example, by allowing the parameters a_0 and a_1 to be estimated, we can investigate whether equating the log realized volatility measure with the log volatility of trend inflation is a sensible thing to do. In particular, if $\log z_t$ is an unbiased measure, we would expect that $a_0 = 0$ and $a_1 = 1$.

In principle one could entertain an additional measurement equation that relates a market-based long-term inflation expectation measure (e.g., the monthly average of the daily breakeven inflation) to the trend inflation π_t^*. In preliminary work we find that the performance of the model deteriorates when this additional source of information is added; there are noticeable discrepancies between the market-based measure and the model-based trend inflation. This likely reflects the fact that breakeven inflation includes not only inflation expectations but also liquidity and inflation risk premiums. While there are a few recent papers, such as Christensen, Lopez, and Rudebusch (2010) and Grishchenko and Huang (2012), that aim to decompose the breakeven inflation into a purely inflation expectations measure and risk premiums, these approaches often involve additional modeling and assumptions about how risk is priced. We therefore do not use the level of the market-based long-term inflation expectations.

This discussion raises the question of the quality of the realized volatility constructed, given that the breakeven inflation includes liquidity and inflation risk premiums. In next section we argue that as long as the risk premiums do not change drastically within a month, the constructed realized volatility should be a reasonable estimate of the underlying volatility. Further, we show that empirically in Section 5 that the constructed realized volatility can refine the model estimates and improve the model fit.

3 Realized Volatility of Inflation Expectations

In this section we discuss how our measure of realized volatility of market-based long-horizon inflation expectations z_t is constructed. These calculations are based on the so-called breakeven inflation or inflation compensation, which is often interpreted as a measure of expected inflation. More specifically, the breakeven inflation is the inflation rate at which the investor receives the same expected return from holding either nominal or inflation-protected bonds. The breakeven inflation reflects expected inflation—and empirical studies (e.g., Gürkaynak, Levin, and Swanson, 2010) typically interpret it as such. But it also reflects compensation investors are demanding for risks associated with the uncertainty about future inflation or liquidity differential between the real and nominal bond markets. One main goal of our paper is to investigate whether the realized volatility of this market-based inflation expectations measure is consistent with its model-based counterpart.

Let $r_t^{(k)}$ represent the real interest rate on a k-period bond and let $i_t^{(k)}$ denote the corresponding nominal interest rate. The breakeven inflation between periods k_1 and k_2 with

$k_2 > k_1$ is calculated as

$$\mathrm{ei}^{(k_1,k_2)} = \frac{k_2(i^{(k_2)} - r^{(k_2)}) - k_1(i^{(k_1)} - r^{(k_1)})}{k_2 - k_1}.$$

This quantity is commonly used as a measure of expected inflation between periods k_1 and k_2. For example, if $k_1 = 2$ and $k_2 = 3$, then $\mathrm{ei}^{(2,3)}$ may be interpreted as the average expected inflation between year 2 and 3. One often uses $\mathrm{ei}^{(5,10)}$ as a measure of long-horizon inflation expectations, although other measures are also used (see, e.g., Jochmann, Koop, and Potter, 2010). In our empirical work, the real and nominal interest rates are the US real and nominal Treasury security yields taken from the Treasury Inflation-Protected Securities (TIPS) market. For further discussion of the TIPS market, see, e.g., Potter and Rosenberg (2007). We use these daily long-horizon inflation expectations to construct the relevant realized volatility.

The construction of realized volatility for inflation expectations is complicated by the fact that this market-based measure of inflation expectations does not only reflect inflation expectations, but also includes risk premiums associated with future inflation uncertainty as well as the difference in liquidity between the nominal and real bond markets. Hence, using the *level* of breakeven inflation as a measure of inflation expectations might be problematic. However, if such risk premiums are constant within a month, they can be removed by using demeaned data.

Specifically, let $\mathrm{ei}_{t,i}^{(k_1,k_2)}$ denote the breakeven inflation on the i-th day in month t and write

$$\mathrm{ei}_{t,i}^{(k_1,k_2)} = \pi_{t,i}^{(k_1,k_2)} + \phi_t,$$

where $\pi_{t,i}^{(k_1,k_2)}$ is the expected average inflation between periods k_1 and k_2 given the information on the i-th day in month t and ϕ_t is a risk premium term that is independent of i. Then, it is easy to see that $\mathrm{ei}_{t,i}^{(k_1,k_2)} - \overline{\mathrm{ei}}_t^{(k_1,k_2)}$ is independent of ϕ_t, where $\overline{\mathrm{ei}}_t^{(k_1,k_2)}$ is the average of the daily observations within month t.

Therefore, we construct the realized volatility by using the demeaned quadratic variation as

$$z_t = \frac{1}{n_t} \sum_{i=1}^{n_t} \left(\mathrm{ei}_{t,i}^{(k_1,k_2)} - \overline{\mathrm{ei}}_t^{(k_1,k_2)} \right)^2,$$

where n_t is the number of daily observations in month t. As mentioned, this demeaned quadratic variation is used to improve efficiency under the assumption that the risk premiums are constant in any month t, whereas they are allowed to change across different months. Of course whether this assumption is reasonable or not is an empirical question. And our results in the next section show that the realized volatility thus constructed helps refine the estimates.

In the context of high-frequency financial data (e.g., stock returns observed every 5 minutes), the quadratic variation is a simple estimator for the daily volatility that has good properties. In particular, since the quadratic variation of continuous finite-variation process is zero (see, e.g., property (ii) of Proposition 2 in Andersen et al., 2003), the mean component becomes irrelevant for the quadratic variation. Hence, in principle we do not need to demean the series, because the variation contributed by the mean component is an order of magnitude smaller than the variation contributed by the volatility. However, these results are based on infill asymptotics which might not apply in our context as there are on average only 22 daily observations in each month.[1] Therefore, in the measurement equation (8) for z_t, we allow for the potential bias associated with this measurement problem.

4 Data and Estimation

In this section we describe the data source and outline the posterior sampler. The daily real and nominal interest rates used to compute our measure of realized volatility of inflation expectations are the US real and nominal Treasury security yields taken from the Treasury Inflation-Protected Securities (TIPS) market. Since bond yields from the TIPS market are only available relatively recently, we restrict our sample to January 2003 to December 2015. We use annualized US monthly CPI inflation rate as our inflation measure. In our baseline results, we use the maturities of 5 and 10 years. That is, the breakeven inflation is $\mathrm{ei}^{(5,10)}$. All data are sourced from the Federal Reserve Bank of St. Louis economic database. In Appendix B we report results based on the breakeven inflation $\mathrm{ei}^{(7,10)}$. The estimation results are broadly similar to the baseline case.

The model (4)–(8) is estimated using Markov chain Monte Carlo (MCMC) methods and the details are delineated in Appendix A. To summarize the posterior sampler, the parameters and the latent states are partitioned into five blocks:

[1] In addition to the quadratic variation discussed above, other realized volatility construction methods such as bipower variation (Barndorff-Nielsen and Shephard, 2004), flat-top realized kernel (Barndorff-Nielsen et al., 2008) and non-negative realized kernel (Barndorff-Nielsen et al., 2011) are proposed to deal with jumps and microstructure noise. All these methods rely on infill asymptotics, and no simple small sample properties are available to the best of our knowledge.

1. $\mathbf{g} = (g_1, \ldots, g_T)'$ is the vector of log volatility of the trend inflation;

2. $\mathbf{h} = (h_1, \ldots, h_T)'$ is the vector of log volatility of the transitory component of inflation;

3. $\boldsymbol{\pi}^* = (\pi_1^*, \ldots, \pi_T^*)'$ is the vector of trend inflation;

4. $(\sigma_g^2, \sigma_h^2, \sigma_z^2)$ is the collection of the error variances;

5. $\mathbf{a} = (a_0, a_1)'$ is the vector of regression coefficients in the inflation expectation uncertainty equation (8).

Each block of parameters is simulated conditional on the other blocks. After discarding a burn-in sample, the sample of the random draws is used for inference. We refer the readers to Appendix A for technical details.

5 Empirical Results

Before presenting results from the model (4)–(8), we conduct a preliminary analysis to assess how well our measure of realized volatility of long-horizon inflation expectations matches some conventional estimates of inflation expectations uncertainty. To that end, we estimate the UCSV model of Stock and Watson (2007) using only inflation data, and report the estimates of the log volatility corresponding to the trend inflation. We also plot the log realized volatility constructed by using the real and nominal bond yields as discussed in Section 3. The results are reported in Figure 1.

Not surprisingly, the realized volatility is much nosier than the trend stochastic volatility estimates. There is a clear comovement between these two quantities, though they diverge in certain episodes. For example, the realized volatility sharply increases in 2008-2009 at the onset of the Great Recession. While the trend stochastic volatility estimate increases as well in that period, the rise is not as large. These observations suggest that the realized volatility measure is potentially useful in providing additional information for estimating the inflation expectations uncertainty, but it might be inappropriate to treat the realized volatility as an unbiased estimate of the trend stochastic volatility.

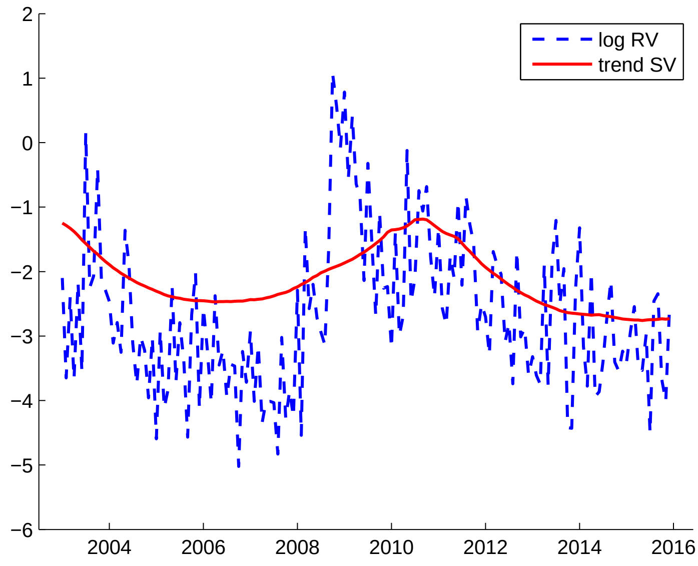

Figure 1: The log realized volatility measure and the log volatility estimates corresponding to the trend inflation under the UCSV model.

5.1 Inflation Expectations Uncertainty

We first report the stochastic volatility estimates from the model (4)–(8). That is, we extend the UCSV model of Stock and Watson (2007) by incorporating the realized volatility measure z_t as specified in (8). We refer to this model as UCSV-RV. For comparison we also present the estimates from the UCSV model. The results are depicted in Figure 2.

The most noticeable difference between the estimates from the two models occurs at the onset of the Great Recession in 2008. Under UCSV the inflation expectations uncertainty exhibits slow and gradual movements, and peaks only in 2011. This is intuitive as the UCSV model uses only inflation data; under this model the inflation expectations volatility is assumed to change slowly as specified in (7). Hence, by construction the UCSV gives smooth estimates of inflation expectations uncertainty.

In contrast, the inflation expectations uncertainty under UCSV-RV peaks in the first quarter of 2009, reflecting both the sudden and drastic drop of inflation, as well as the surge of realized volatility in late 2008 and early 2009. By incorporating the realized volatility measure, the UCSV-RV model is able to pick up drastic changes in the inflation expectations uncertainty.

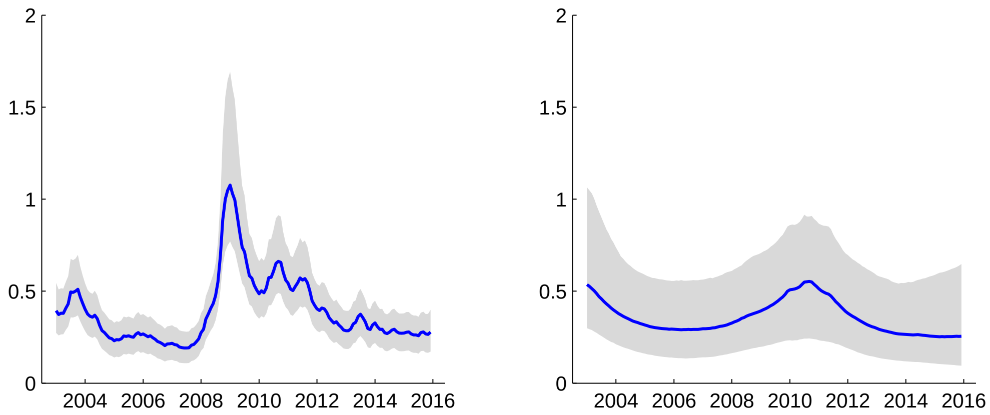

Figure 2: Stochastic volatility estimates expressed in standard deviations $\exp(g_t/2)$ from UCSV-RV (left panel) and UCSV (right panel). The shaded areas represent the 16- and 84-percentiles.

In addition, Figure 2 also shows the associated 68% credible intervals of the estimates.[2] Since the UCSV-RV model incorporates more information, in general the volatility of the inflation expectations is estimated more precisely compared to UCSV, as evidenced by the typically narrower credible intervals. For example, the 68% credible interval under UCSV in December 2015 is about 2.5 times wider than that of UCSV-RV.

The only exception occurs at the onset of the Great Recession, when the credible intervals of UCSV-RV become noticeably larger. This increase in uncertainty is due to the conflict between two sources of information: the realized volatility measure increases markedly at the onset of the Great Recession, whereas the state equation (7) dictates a smooth evolution of the inflation expectations volatility. Even though the former source of information dominates the posterior estimates, the model registers a higher level of parameter uncertainty.

Overall, the results show that adding the information in the realized volatility measure changes the estimates of inflation expectations uncertainty. Moreover, this additional information typically refines the inflation expectations uncertainty estimates.

[2]Under the normal distribution, the probability within plus or minus one standard deviation from the mean is about 0.68.

5.2 Inflation Expectations

To investigate how the inclusion of realized volatility measure affects the trend inflation estimates, we report in Figure 3 the corresponding estimates from UCSV-RV. For comparison, we also present estimates from a version that also incorporates the level of breakeven inflation. Specifically, let x_t denote the average breakeven inflation in month t. We then augment our model with the additional measurement equation:

$$x_t = b_0 + b_1\pi_t^* + u_t^x, \quad u_t^x \sim \mathcal{N}(0, \sigma_x^2). \tag{9}$$

We call this variant UCSV-RV-BE. Finally, we also report results from the UCSV model of Stock and Watson (2007).

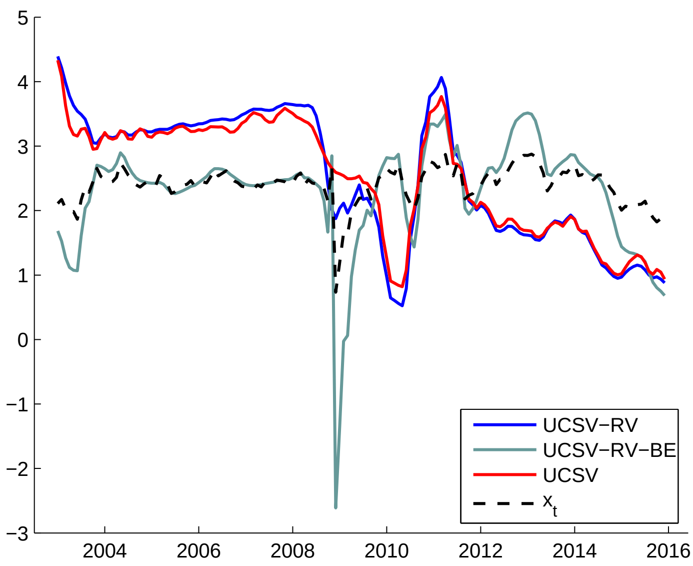

Figure 3: Trend estimates from UCSV-RV, UCSV-RV-BE and UCSV.

Comparing the estimates from UCSV-RV and UCSV, it is clear that they are remarkably similar in most of the sample. The exception is the few months in the early phase of the Great Recession. For example, in November 2008 the trend estimate under UCSV-RV is about 1.9%, whereas that under UCSV is 2.6%. This reflects the heightened trend inflation or inflation expectations uncertainty under UCSV-RV, and the model gives more weight to the actual inflation data. Since the inflation rate then was negative, this drags down the trend inflation estimate.

Interestingly, the estimates under UCSV-RV-BE become negative during the Great Recession, partly due to the sudden drop of the breakeven inflation. These results confirm the conclusion in Faust and Wright (2013), who warn against interpreting the breakeven inflation as a pure measure of inflation expectations. In addition, we show below that UCSV-RV-BE fits the inflation data relatively poorly compared to the other two models.

5.3 Parameter Estimates

In this section we report the posterior estimates of a few parameters of interest to highlight the properties of the proposed model in (4)–(8). One main question we wish to address is: Is the realized volatility measure a useful estimate of the underlying inflation expectations uncertainty? To answer that question, we plot the prior and posterior densities of a_0 and a_1 in Figure 4. Recall that a_0 and a_1 are respectively the intercept and slope of the log realized volatility equation in (8). If the log realized volatility is an unbiased measure of the underlying inflation expectations uncertainty, we expect $a_0 = 0$ and $a_1 = 1$.

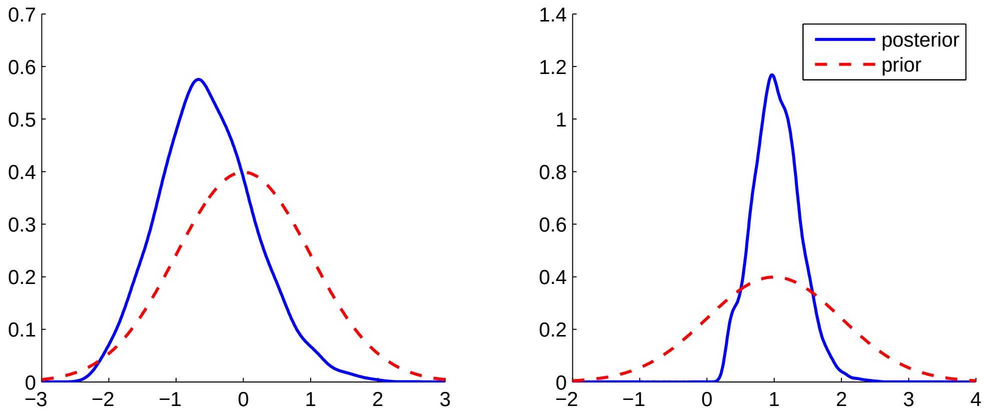

Figure 4: Prior and posterior densities of a_0 (left panel) and a_1 (right panel).

The left panel of Figure 4 shows that the posterior density of a_0 is centered around -0.5, suggesting some evidence of bias. However, the parameter uncertainty is sufficiently large that the density has substantial mass around 0. In fact, the Bayes factor in favor of the hypothesis $a_0 = 0$ is 0.98, showing that equal evidence in favor and against $a_0 = 0$.

In contrast, the posterior density of a_1 is centered around 1 and has more mass around that value compared to the prior. The Bayes factor in favor of the hypothesis $a_1 = 1$ is 2.9, indicating some evidence that $a_1 = 1$. Overall, we conclude that the realized volatility provides useful information for estimating the inflation expectations uncertainty, but it may not be a perfectly unbiased measure.

Next, we report in Table 1 the posterior estimates of σ_z^2, the error variance in the log realized volatility equation. The posterior mean of σ_z^2 is 0.66, compared to the prior mean of 0.3. This variance estimate is relatively large, suggesting that the realized volatility is a noisy measure of the underlying inflation expectations uncertainly. This is also apparent in Figure 1.

Table 1: Posterior estimates of selected parameters.

parameter	a_0	a_1	σ_z^2
mean	-0.53	1.04	0.66
std. dev.	(0.71)	(0.35)	(0.09)

5.4 Model Comparison

Lastly, we provide some evidence that the proposed model is favored by the data. Specifically, we compute the marginal likelihood for UCSV, UCSV-RV and UCSV-RV-BE. Each marginal likelihood estimate is computed by decomposing the marginal density of the inflation data as the product of predictive likelihoods. In particular, let $\boldsymbol{\pi}_{1:t} = (\pi_1, \ldots, \pi_t)'$ denote the inflation data up to time t. Then, we can factor the marginal likelihood for model M_k as follows:

$$p(\boldsymbol{\pi} \,|\, \mathbf{W}_k, M_k) = p(\pi_1 \,|\, \mathbf{W}_{1,k}, M_k) \prod_{t=1}^{T-1} p(\pi_{t+1} \,|\, \boldsymbol{\pi}_{1:t}, \mathbf{W}_{1:t,k}, M_k),$$

where $p(\pi_{t+1} \,|\, \boldsymbol{\pi}_{1:t}, \mathbf{W}_{1:t,k}, M_k)$ is the predictive likelihood and $\mathbf{W}_{1:t,k}$ is the set of additional data up to time t used in model M_k (e.g., the realized volatility or the level of breakeven inflation).

Therefore, even though some models use more than inflation data, the marginal likelihoods thus computed are comparable across models. The results are reported in Table 2.

Table 2: Log marginal likelihood estimates of selected models.

	UCSV	UCSV-RV	UCSV-RV-BE
log marginal likelihood	-415	-412	-423

Our baseline model UCSV-RV is the best among the three models, showing that adding the realized volatility measure improves the model fit relative to the increase in model complexity. For example, the Bayes factor in favor of UCSV-RV against UCSV is about 20 ($\approx e^3$). In other words, if we assume both models are equally probable *a priori*, the former is 20 times more likely than the latter given the data. Interestingly, even though the UCSV-RV-BE model has the most information, its performance is worse than even UCSV. This result suggests that there are large discrepancies between the breakeven inflation and the model-based trend inflation.

6 Concluding Remarks and Future Research

We use daily breakeven inflation to construct a realized measure of inflation expectations volatility. We then incorporate this realized volatility measure in a standard UCSV model to investigate how well it matches the model-based measure of trend inflation volatility. We find that the two measure are largely compatible, and the realized volatility helps refine the estimates of inflation expectations uncertainty. In addition, we find significant changes of inflation expectations uncertainty during the Great Recession.

In future work, it would be fruitful to have a term structure model to combine multiple realized volatility measures from different horizons. It would also be interesting in future work to study the impact of inflation expectations uncertainty on other macroeconomic variables.

A Appendix: Estimation Details

In this appendix we provide the estimation details of fitting the UCSV-RV-BE model defined in (4)–(9) using MCMC methods. The UCSV-RV model is a restricted version where equation (9) is omitted. We implement a Gibbs sampler that sequentially draws from the full conditional distributions of the parameters and the latent states. The parameters are $\sigma^2_h, \sigma^2_g, \sigma^2_z, \sigma^2_x,$, $\mathbf{a} = (a_0, a_1)'$ and $\mathbf{b} = (b_0, b_1)'$, and the latent states are $\mathbf{g}$, $\mathbf{h}$ and $\boldsymbol{\pi}^*$.

Let $\mathcal{IG}(c_1, c_2)$ denote the inverse-gamma distribution with mean $c_1/(c_2 - 1)$. We consider the following priors: $\sigma^2_h \sim \mathcal{IG}(s_h/2, v_h/2)$, $\sigma^2_g \sim \mathcal{IG}(s_g/2, v_g/2)$, $\sigma^2_z \sim \mathcal{IG}(s_z/2, v_z/2)$, $\sigma^2_x \sim \mathcal{IG}(s_x/2, v_x/2)$, $\mathbf{a} \sim \mathcal{N}(\mathbf{m}_a, \mathbf{V}_a)$ and $\mathbf{b} \sim \mathcal{N}(\mathbf{m}_b, \mathbf{V}_b)$. Finally, we initialize the state equations using $g_1 \sim \mathcal{N}(m_g, V_g)$, $h_1 \sim \mathcal{N}(m_h, V_h)$ and $(\pi^*_1 \,|\, g_1) \sim \mathcal{N}(m_{\pi^*}, \mathrm{e}^{g_1} V_{\pi^*})$.

Posterior draws can be obtained by sequentially performing the following MCMC steps:

1. **Sample** $\mathbf{g} = (g_1, \ldots, g_T)'$.

To sample the log volatilities, we adopt the auxiliary mixture sampler of Kim, Shepherd, and Chib (1998) by approximating the nonlinear state space model using a mixture of conditionally linear Gaussian state space models with mixture indicators $\mathbf{s}^g = (s^g_1, \ldots, s^g_T)'$. We first sample the mixture indicators $\mathbf{s}^g$ given the current g. Then, we draw g given the mixture indicators $\mathbf{s}^g$. For a textbook treatment of the auxiliary mixture sampler, see Chan and Hsiao (2014).

First define $y^g_1 = \log((\pi^*_1 - m_{\pi^*})^2/V_{\pi^*})$ and $y^g_t = \log((\pi^*_t - \pi^*_{t-1})^2)$ for $t = 2, \ldots, T$. Since $s^g_1, \ldots, s^g_T$ are conditionally independent, we can sample them sequentially. Each s^g_t takes values in $\{1, \ldots, 7\}$ with probabilities

$$p(s^g_t = k \,|\, y^g_t, g_t) \propto w_k f_N(y^g_t - g_t; m_k, v^2_k) \quad \text{for } k = 1, 2, \ldots, 7,$$

where $f_N(\cdot; u, v^2)$ is the Gaussian density with mean u and variance v^2, and the values (w_k, m_k, v^2_k) are given in Table 4 of Kim, Shepherd, and Chib (1998).

Next, we sample $\mathbf{g}$ given the mixture indicators $\mathbf{s}^g$. To that end, stack $\mathbf{y}^g = (y^g_1, \ldots, y^g_T)'$ and rewrite (5) as

$$\mathbf{y}^g = \mathbf{g} + \boldsymbol{\varepsilon}^g, \quad \boldsymbol{\varepsilon}^g \sim \mathcal{N}(\mathbf{d}_{\mathbf{s}^g}, \boldsymbol{\Omega}_{\mathbf{s}^g})$$

with density function

$$p(\mathbf{y}^g \mid \mathbf{g}, \mathbf{s}^g) \propto \exp\left(-\frac{1}{2}(\mathbf{y}^g - \mathbf{d}_{\mathbf{s}^g} - \mathbf{g})'\boldsymbol{\Omega}_{\mathbf{s}^g}^{-1}(\mathbf{y}^g - \mathbf{d}_{\mathbf{s}^g} - \mathbf{g})\right),$$

where $\mathbf{d}_{\mathbf{s}^g}$ and $\boldsymbol{\Omega}_{\mathbf{s}^g}$ are constant matrices determined by $\mathbf{s}^g$, and $\boldsymbol{\Omega}_{\mathbf{s}^g}$ is diagonal.

Next, write the state equation (7) in matrix form

$$\mathbf{H}\mathbf{g} = \mathbf{u}^g, \quad \mathbf{u}^g \sim \mathcal{N}(\widetilde{\mathbf{m}}_g, \mathbf{S}_g),$$

where $\widetilde{\mathbf{m}}_g = (m_g, 0, 0, \ldots, 0)'$, $\mathbf{S}_g = \mathrm{diag}(V_g, \sigma_g^2, \ldots, \sigma_g^2)$ and

$$\mathbf{H} = \begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \\ -1 & 1 & 0 & \cdots & 0 \\ 0 & -1 & 1 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & -1 & 1 \end{pmatrix}.$$

Hence, the prior density of $\mathbf{g}$ is given by

$$\begin{aligned} p(\mathbf{g} \mid \sigma_g^2) &\propto \exp\left(-\frac{1}{2}(\mathbf{g} - \mathbf{H}^{-1}\widetilde{\mathbf{m}}_g)'\mathbf{H}'\mathbf{S}_g^{-1}\mathbf{H}(\mathbf{g} - \mathbf{H}^{-1}\widetilde{\mathbf{m}}_g)\right) \\ &= \exp\left(-\frac{1}{2}(\mathbf{g} - \mathbf{1}_T\mathbf{m}_g)'\mathbf{H}'\mathbf{S}_g^{-1}\mathbf{H}(\mathbf{g} - \mathbf{1}_T\mathbf{m}_g)\right), \end{aligned}$$

where $\mathbf{1}_T$ is a $T \times 1$ column of ones. Note that the prior mean of $\mathbf{g}$ is $\mathbf{1}_T m_g$ since g_t is a random walk process.

Now, let $\widetilde{\mathbf{z}} = (\log z_1, \ldots, \log z_T)'$ and rewrite (8) as

$$\widetilde{\mathbf{z}} = a_0\mathbf{1}_T + a_1\mathbf{g} + \mathbf{u}^z, \quad \mathbf{u}^z \sim \mathcal{N}(\mathbf{0}, \sigma_z^2\mathbf{I}_T)$$

with density function

$$p(\widetilde{\mathbf{z}} \mid \mathbf{g}, \sigma_z^2, a_0, a_1) \propto \exp\left(-\frac{1}{2\sigma_z^2}(\widetilde{\mathbf{z}} - a_0\mathbf{1}_T - a_1\mathbf{g})'(\widetilde{\mathbf{z}} - a_0\mathbf{1}_T - a_1\mathbf{g})\right).$$

Finally, the full conditional density of $\mathbf{g}$ is given by

$$\begin{aligned}
p(\mathbf{g}\,|\,\mathbf{y}^g, \widetilde{\mathbf{z}}, \mathbf{s}^g, \sigma_g^2, \sigma_z^2, a_0, a_1) &\propto p(\mathbf{y}^g\,|\,\mathbf{g}, \mathbf{s}^g)p(\mathbf{g}\,|\,\sigma_g^2)p(\widetilde{\mathbf{z}}\,|\,\mathbf{g}, \sigma_z^2, a_0, a_1) \\
&\propto \exp\left\{-\frac{1}{2}\left[\mathbf{g}'\left(\boldsymbol{\Omega}_{\mathbf{s}^g}^{-1} + \mathbf{H}'\mathbf{S}_g^{-1}\mathbf{H} + \frac{a_1^2}{\sigma_z^2}\mathbf{I}_T\right)\mathbf{g}\right.\right. \\
&\qquad \left.\left. -2\mathbf{g}'\left(\boldsymbol{\Omega}_{\mathbf{s}^g}^{-1}(\mathbf{y}^g - \mathbf{d}_{\mathbf{s}^g}) + \mathbf{H}'\mathbf{S}_g^{-1}\mathbf{H}\mathbf{1}_T m_g + \frac{a_1}{\sigma_z^2}(\widetilde{\mathbf{z}} - a_0\mathbf{1}_T)\right)\right]\right\},
\end{aligned}$$

which is the kernel of the $\mathcal{N}(\widehat{\mathbf{g}}, \mathbf{K}_{\mathbf{g}}^{-1})$ distribution, where

$$\begin{aligned}
\mathbf{K}_{\mathbf{g}} &= \boldsymbol{\Omega}_{\mathbf{s}^g}^{-1} + \mathbf{H}'\mathbf{S}_g^{-1}\mathbf{H} + \frac{a_1^2}{\sigma_z^2}\mathbf{I}_T, \\
\widehat{\mathbf{g}} &= \mathbf{K}_{\mathbf{g}}^{-1}\left(\boldsymbol{\Omega}_{\mathbf{s}^g}^{-1}(\mathbf{y}^g - \mathbf{d}_{\mathbf{s}^g}) + \mathbf{H}'\mathbf{S}_g^{-1}\mathbf{H}\mathbf{1}_T m_g + \frac{a_1}{\sigma_z^2}(\widetilde{\mathbf{z}} - a_0\mathbf{1}_T)\right).
\end{aligned}$$

Notice that $\mathbf{H}'\mathbf{S}_{\mathbf{g}}^{-1}\mathbf{H}\mathbf{1}_T m_g = (V_g^{-1}m_g, 0, 0, \ldots, 0)'$. Since $\mathbf{K}_{\mathbf{g}}$ is a band precision matrix, the precision sampler in Chan and Jeliazkov (2009) can be used to sample $\mathbf{g}$ efficiently.

2. Sample $\mathbf{h} = (h_1, \ldots, h_T)'$.

Similar to the previous step, we implement the auxiliary mixture sampler of Kim, Shepherd, and Chib (1998) by first drawing the mixture indicators $\mathbf{s}^h = (s_1^h, \ldots, s_T^h)'$ given $\mathbf{h}$ and other parameters, followed by sampling $\mathbf{h}$ given the mixture indicators $\mathbf{s}^h$. To that end, define $y_t^h = \log\left((\pi_t - \pi_t^*)^2\right)$ and stack $\mathbf{y}^h = (y_1^h, \ldots, y_T^h)'$. Then, each s_t^h can be drawn from the conditional posterior distribution as before.

Next, rewrite (4) in matrix form:

$$\mathbf{y}^h = \mathbf{h} + \boldsymbol{\varepsilon}^h, \quad \boldsymbol{\varepsilon}^h \sim \mathcal{N}(\mathbf{d}_{\mathbf{s}^h}, \boldsymbol{\Omega}_{\mathbf{s}^h}),$$

where $\mathbf{d}_{s^h}$ and $\boldsymbol{\Omega}_{\mathbf{s}^h}$ are constant matrices determined by $\mathbf{s}^h$. Similarly, rewrite (6) as

$$\mathbf{H}\mathbf{h} = \mathbf{u}^h, \quad \mathbf{u}^h \sim \mathcal{N}(\widetilde{\mathbf{m}}_h, \mathbf{S}_h),$$

where $\widetilde{\mathbf{m}}_h = (m_h, 0, 0, \ldots, 0)'$ and $\mathbf{S}_h = \mathrm{diag}(V_h, \sigma_h^2, \ldots, \sigma_h^2)$. Using a similar derivation as in Step 1, we have

$$(\mathbf{h}\,|\,\mathbf{y}^h, \mathbf{s}^h, \sigma_h^2) \sim \mathcal{N}(\widehat{\mathbf{h}}, \mathbf{K}_{\mathbf{h}}^{-1}),$$

where $\mathbf{K}_{\mathbf{h}} = \boldsymbol{\Omega}_{\mathbf{s}^h}^{-1} + \mathbf{H}'\mathbf{S}_h^{-1}\mathbf{H}$ and $\widehat{\mathbf{h}} = \mathbf{K}_h^{-1}\left(\boldsymbol{\Omega}_{\mathbf{s}^h}^{-1}(\mathbf{y}^h - \mathbf{d}_{\mathbf{s}^h}) + \mathbf{H}'\mathbf{S}_h^{-1}\mathbf{H}\mathbf{1}_T m_h\right)$.

3. Sample $\boldsymbol{\pi}^* = (\pi_1^*, \ldots, \pi_T^*)'$.

Information about $\boldsymbol{\pi}^*$ comes from three sources: the two measurement equations (4) and (9), as well as the state equation (5). First, rewrite the three equations as

$$
\begin{aligned}
\boldsymbol{\pi} &= \boldsymbol{\pi}^* + \mathbf{u}^{\pi}, & \mathbf{u}^{\pi} &\sim \mathcal{N}(\mathbf{0}, \mathbf{S}_{\pi}),\\
\mathbf{x} &= b_0\mathbf{1}_T + b_1\boldsymbol{\pi}^* + \mathbf{u}^{x}, & \mathbf{u}^{x} &\sim \mathcal{N}(\mathbf{0}, \sigma_x^2\mathbf{I}_T),\\
\mathbf{H}\boldsymbol{\pi}^* &= \mathbf{u}^{\pi^*}, & \mathbf{u}^{\pi^*} &\sim \mathcal{N}(\widetilde{\mathbf{m}}_{\pi^*}, \mathbf{S}_{\pi^*}),
\end{aligned}
$$

where $\mathbf{S}_{\pi} = \mathrm{diag}(\mathrm{e}^{h_1}, \ldots, \mathrm{e}^{h_T})$, $\widetilde{\mathbf{m}}_{\pi^*} = (m_{\pi^*}, 0, \ldots, 0)'$ and $\mathbf{S}_{\pi^*} = \mathrm{diag}(\mathrm{e}^{g_1}V_{\pi^*}, \mathrm{e}^{g_2}, \ldots, \mathrm{e}^{g_T})$. Using a similar derivation as in Step 1, we have

$$(\boldsymbol{\pi}^* \,|\, \boldsymbol{\pi}, \mathbf{g}, \mathbf{h}, \mathbf{x}, \mathbf{b}) \sim \mathcal{N}(\widehat{\boldsymbol{\pi}}^*, \mathbf{K}_{\boldsymbol{\pi}^*}^{-1}),$$

where

$$\mathbf{K}_{\boldsymbol{\pi}^*} = \mathbf{H}'\mathbf{S}_{\pi^*}^{-1}\mathbf{H} + \mathbf{S}_{\pi}^{-1} + \frac{b_1^2}{\sigma_x^2}\mathbf{I}_T, \quad \widehat{\boldsymbol{\pi}}^* = \mathbf{K}_{\boldsymbol{\pi}^*}^{-1}\left(\mathbf{H}'\mathbf{S}_{\pi^*}^{-1}\widetilde{\mathbf{m}}_{\pi^*} + \mathbf{S}_{\pi}^{-1}\boldsymbol{\pi} + \frac{b_1}{\sigma_x^2}(\mathbf{x} - \mathbf{1}_T b_0)\right).$$

Note that $\mathbf{H}'\mathbf{S}_{\pi^*}^{-1}\widetilde{\mathbf{m}}_{\boldsymbol{\pi}^*} = (m_{\pi^*}/(\mathrm{e}^{g_1}V_{\pi^*}), 0, \ldots, 0)'$. Again $\mathbf{K}_{\boldsymbol{\pi}^*}$ is a band precision matrix, we use the algorithm in Chan and Jeliazkov (2009) to sample $\boldsymbol{\pi}^*$.

4. Sample $\sigma_g^2, \sigma_h^2, \sigma_z^2$ and σ_x^2.

This step is standard as $\sigma_g^2, \sigma_h^2, \sigma_z^2$ and σ_x^2 are conditionally independent and each follows an inverse-gamma distribution. Define $\Delta\mathbf{g} = (g_2 - g_1, g_3 - g_2, \ldots, g_T - g_{T-1})'$, $\Delta\mathbf{h} = (h_2 - h_1, h_3 - h_2, \ldots, h_T - h_{T-1})'$, $\mathbf{u}^z = \widetilde{\mathbf{z}} - a_0\mathbf{1}_T - a_1\mathbf{g}$ and $\mathbf{u}^x = \mathbf{x} - b_0\mathbf{1}_T - b_1\boldsymbol{\pi}^*$. Then, we have

$$
\begin{aligned}
\sigma_h^2 &\sim \mathcal{IG}(\widehat{s}_h/2, \widehat{v}_h/2),\\
\sigma_g^2 &\sim \mathcal{IG}(\widehat{s}_g/2, \widehat{v}_g/2),\\
\sigma_z^2 &\sim \mathcal{IG}(\widehat{s}_z/2, \widehat{v}_z/2),\\
\sigma_x^2 &\sim \mathcal{IG}(\widehat{s}_x/2, \widehat{v}_x/2),
\end{aligned}
$$

where

$$
\begin{aligned}
\widehat{s}_h &= s_h + (\Delta\mathbf{h})'\Delta\mathbf{h} & \widehat{v}_h &= v_h + T - 1,\\
\widehat{s}_g &= s_g + (\Delta\mathbf{g})'\Delta\mathbf{g} & \widehat{v}_g &= v_g + T - 1,\\
\widehat{s}_z &= s_z + (\mathbf{u}^z)'\mathbf{u}^z & \widehat{v}_z &= v_z + T,\\
\widehat{s}_x &= s_x + (\mathbf{u}^x)'\mathbf{u}^x & \widehat{v}_x &= v_x + T.
\end{aligned}
$$

5. Sample $\mathbf{a} = (a_0, a_1)'$.

This step is standard since $(\mathbf{a} \,|\, z, g, \sigma_z^2) \sim \mathcal{N}(\widehat{\mathbf{a}}, \mathbf{K}_{\mathbf{a}}^{-1})$, where

$$\mathbf{K}_{\mathbf{a}} = \mathbf{V}_a^{-1} + \frac{1}{\sigma_z^2}\mathbf{X}_a'\mathbf{X}_a, \quad \widehat{\mathbf{a}} = \mathbf{K}_{\mathbf{a}}^{-1}\left(\mathbf{V}_a^{-1}\mathbf{m}_a + \frac{1}{\sigma_z^2}\mathbf{X}_a'\widetilde{\mathbf{z}}\right)$$

with $\widetilde{\mathbf{z}} = (\log z_1, \ldots, \log z_T)'$ and $\mathbf{X}_a = [\mathbf{1}_T, \mathbf{g}]$ is constructed by stacking the regressors.

6. Sample $\mathbf{b} = (b_0, b_1)'$.

Similarly, let $\mathbf{X}_b = [\mathbf{1}_T, \boldsymbol{\pi}^*]$. Then, we have $(\mathbf{b} \,|\, \mathbf{x}, \boldsymbol{\pi}^*, \sigma_x^2) \sim \mathcal{N}(\widehat{\mathbf{b}}, \mathbf{K}_{\mathbf{b}}^{-1})$, where

$$\mathbf{K}_{\mathbf{b}} = \mathbf{V}_b^{-1} + \frac{1}{\sigma_x^2}\mathbf{X}_b'\mathbf{X}_b, \quad \widehat{\mathbf{1}} = \mathbf{K}_{\mathbf{b}}^{-1}\left(\mathbf{V}_b^{-1}\mathbf{m}_b + \frac{1}{\sigma_x^2}\mathbf{X}_b'\mathbf{x}\right).$$

B Appendix: Additional Results

In this appendix we re-estimate the models using another measure of realized volatility. Specifically, the realized volatility is computed from the the breakeven inflation $ei^{(7,10)}$. The results are reported in Figure 5 to Figure 7. The conclusions we draw from these results are the same from the baseline case: the realized volatility provides useful information for the underlying inflation expectations uncertainty and typically helps refine the estimates.

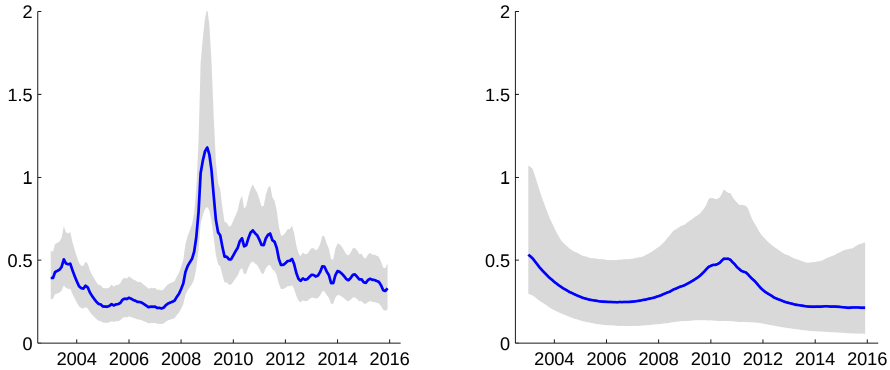

Figure 5: Stochastic volatility estimates expressed in standard deviations $\exp(g_t/2)$ from UCSV-RV (left panel) and UCSV (right panel). The shaded areas represent the 16- and 84-percentiles. The realized volatility is computed from the the breakeven inflation $ei^{(7,10)}$.

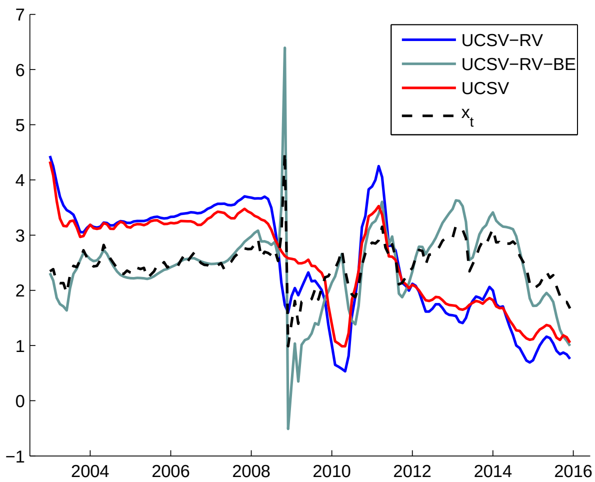

Figure 6: Trend estimates from UCSV-RV, UCSV-RV-BE and UCSV. The realized volatility is computed from the the breakeven inflation $ei^{(7,10)}$.

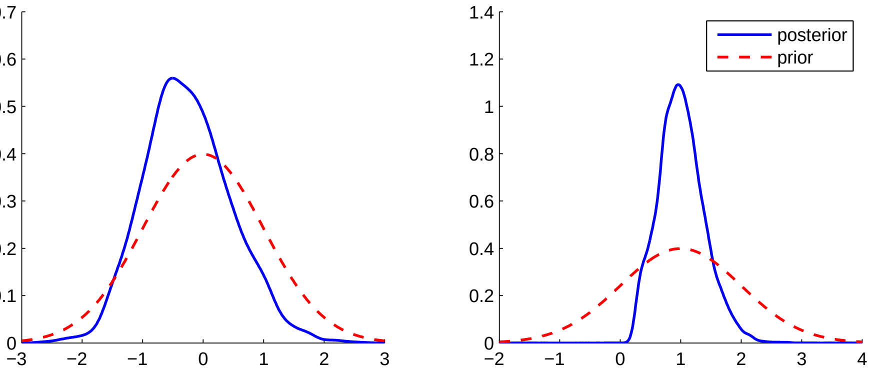

Figure 7: Prior and posterior densities of a_0 (left panel) and a_1 (right panel). The realized volatility is computed from the the breakeven inflation $\mathrm{ei}^{(7,10)}$.

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